

# 1-3

## Measuring Segments

### Common Core State Standards

**G-CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment . . .

**Also G-GPE.B.6**

**MP 2, MP 3, MP 4, MP 6**

**Objective** To find and compare lengths of segments



Analyze the problem to figure out what you know and what you need to find next.

**SOLVE IT!**

**Getting Ready!**

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On a freshwater fishing trip, you catch the fish below. By law, you must release any fish between 15 and 19 in. long. You need to measure your fish, but the front of the ruler on the boat is worn away. Can you keep your fish? Explain how you found your answer.



**MATHEMATICAL PRACTICES**

In the Solve It, you measured the length of an object indirectly.



### Lesson Vocabulary

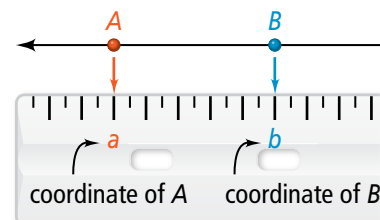
- coordinate
- distance
- congruent segments
- midpoint
- segment bisector

**Essential Understanding** You can use number operations to find and compare the lengths of segments.

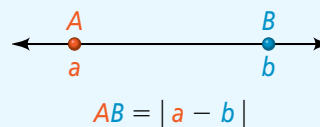
**take note**

### Postulate 1-5 Ruler Postulate

Every point on a line can be paired with a real number. This makes a one-to-one correspondence between the points on the line and the real numbers. The real number that corresponds to a point is called the **coordinate** of the point.



The Ruler Postulate allows you to measure lengths of segments using a given unit and to find distances between points on a number line. Consider  $\overleftrightarrow{AB}$  at the right. The **distance** between points  $A$  and  $B$  is the absolute value of the difference of their coordinates, or  $|a - b|$ . This value is also  $AB$ , or the length of  $\overline{AB}$ .



## Think

What are you trying to find?

$ST$  represents the length of  $\overline{ST}$ , so you are trying to find the distance between points  $S$  and  $T$ .



### Problem 1 Measuring Segment Lengths

What is  $ST$ ?

The coordinate of  $S$  is  $-4$ .

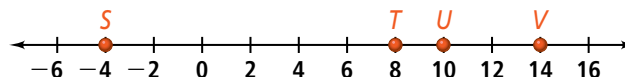
Ruler Postulate

The coordinate of  $T$  is  $8$ .

$$ST = |-4 - 8| \quad \text{Definition of distance}$$

$$= |-12| \quad \text{Subtract.}$$

$$= 12 \quad \text{Find the absolute value.}$$

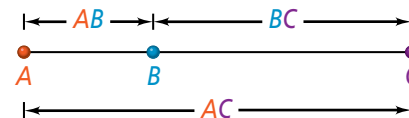


**Got It?** 1. What are  $UV$  and  $SV$  on the number line above?

## Take note

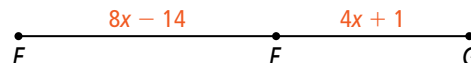
### Postulate 1-6 Segment Addition Postulate

If three points  $A$ ,  $B$ , and  $C$  are collinear and  $B$  is between  $A$  and  $C$ , then  $AB + BC = AC$ .



### Problem 2 Using the Segment Addition Postulate

**Algebra** If  $EG = 59$ , what are  $EF$  and  $FG$ ?



#### Know

$$\begin{aligned} EG &= 59 \\ EF &= 8x - 14 \\ FG &= 4x + 1 \end{aligned}$$

#### Need

$EF$  and  $FG$

#### Plan

Use the Segment Addition Postulate to write an equation.

$$EF + FG = EG \quad \text{Segment Addition Postulate}$$

$$(8x - 14) + (4x + 1) = 59 \quad \text{Substitute.}$$

$$12x - 13 = 59 \quad \text{Combine like terms.}$$

$$12x = 72 \quad \text{Add 13 to each side.}$$

$$x = 6 \quad \text{Divide each side by 12.}$$

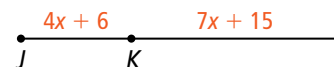
Use the value of  $x$  to find  $EF$  and  $FG$ .

$$EF = 8x - 14 = 8(6) - 14 = 48 - 14 = 34 \quad \text{Substitute 6 for } x.$$

$$FG = 4x + 1 = 4(6) + 1 = 24 + 1 = 25$$

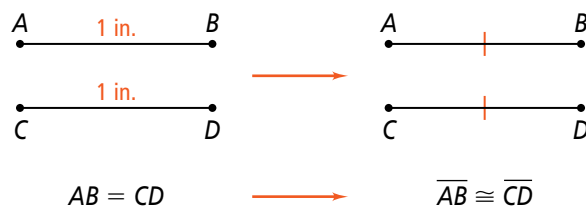


**Got It?** 2. In the diagram,  $JL = 120$ . What are  $JK$  and  $KL$ ?



When numerical expressions have the same value, you say that they are equal ( $=$ ). Similarly, if two segments have the same length, then the segments are **congruent ( $\cong$ ) segments**.

This means that if  $AB = CD$ , then  $\overline{AB} \cong \overline{CD}$ . You can also say that if  $\overline{AB} \cong \overline{CD}$ , then  $AB = CD$ .



As illustrated above, you can mark segments alike to show that they are congruent. If there is more than one set of congruent segments, you can indicate each set with the same number of marks.

## Plan

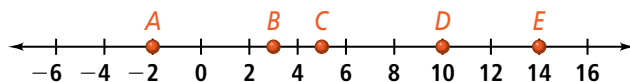
How do you know if segments are congruent?

Congruent segments have the same length. So find and compare the lengths of  $\overline{AC}$  and  $\overline{BD}$ .



### Problem 3 Comparing Segment Lengths

Are  $\overline{AC}$  and  $\overline{BD}$  congruent?



$$AC = |-2 - 5| = |-7| = 7$$

Definition of distance

$$BD = |3 - 10| = |-7| = 7$$

Yes.  $AC = BD$ , so  $\overline{AC} \cong \overline{BD}$ .

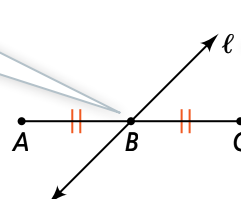


**Got It?** 3. a. Use the diagram above. Is  $\overline{AB}$  congruent to  $\overline{DE}$ ?

b. **Reasoning** To find  $AC$  in Problem 3, suppose you subtract  $-2$  from  $5$ . Do you get the same result? Why?

The **midpoint** of a segment is a point that divides the segment into two congruent segments. A point, line, ray, or other segment that intersects a segment at its midpoint is said to *bisect* the segment. That point, line, ray, or segment is called a **segment bisector**.

$B$  is the midpoint of  $\overline{AC}$ .



$\ell$  is a segment bisector of  $\overline{AC}$ .

## Plan

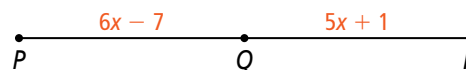
How can you use algebra to solve the problem?

The lengths of the congruent segments are given as algebraic expressions. You can set the expressions equal to each other.



### Problem 4 Using the Midpoint

**Algebra**  $Q$  is the midpoint of  $\overline{PR}$ . What are  $PQ$ ,  $QR$ , and  $PR$ ?



**Step 1** Find  $x$ .

$$\begin{aligned} PQ &= QR && \text{Definition of midpoint} \\ 6x - 7 &= 5x + 1 && \text{Substitute.} \\ x - 7 &= 1 && \text{Subtract } 5x \text{ from each side.} \\ x &= 8 && \text{Add 7 to each side.} \end{aligned}$$

**Step 2** Find  $PQ$  and  $QR$ .

$$\begin{aligned} PQ &= 6x - 7 && QR = 5x + 1 \\ &= 6(8) - 7 && \text{Substitute 8 for } x. && = 5(8) + 1 \\ &= 41 && \text{Simplify.} && = 41 \end{aligned}$$

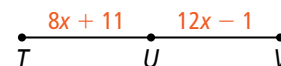
**Step 3** Find  $PR$ .

$$\begin{aligned} PR &= PQ + QR && \text{Segment Addition Postulate} \\ &= 41 + 41 && \text{Substitute.} \\ &= 82 && \text{Simplify.} \end{aligned}$$

$PQ$  and  $QR$  are both 41.  $PR$  is 82.



**Got It?** 4. **a. Reasoning** Is it necessary to substitute 8 for  $x$  in the expression for  $QR$  in order to find  $QR$ ? Explain.  
**b.**  $U$  is the midpoint of  $\overline{TV}$ . What are  $TU$ ,  $UV$ , and  $TV$ ?



## Lesson Check

**Do you know HOW?**

Name each of the following.



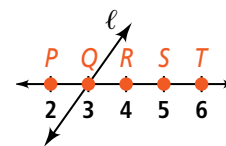
- The point on  $\overline{DA}$  that is 2 units from  $D$
- Two points that are 3 units from  $D$
- The coordinate of the midpoint of  $\overline{AG}$
- A segment congruent to  $\overline{AC}$

**Do you UNDERSTAND?**



**MATHEMATICAL PRACTICES**

- Vocabulary** Name two segment bisectors of  $\overline{PR}$ .
- Compare and Contrast** Describe the difference between saying that two segments are *congruent* and saying that two segments have *equal length*. When would you use each phrase?
- Error Analysis** You and your friend live 5 mi apart. He says that it is 5 mi from his house to your house and  $-5$  mi from your house to his house. What is the error in his argument?





# Practice and Problem-Solving Exercises



## A Practice

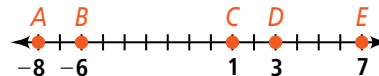
Find the length of each segment.

8.  $\overline{AB}$

9.  $\overline{BD}$

10.  $\overline{AD}$

11.  $\overline{CE}$



See Problem 1.

Use the number line at the right for Exercises 12–14.

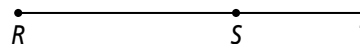
12. If  $RS = 15$  and  $ST = 9$ , then  $RT = \square$ .

13. If  $ST = 15$  and  $RT = 40$ , then  $RS = \square$ .

14. **Algebra**  $RS = 8y + 4$ ,  $ST = 4y + 8$ , and  $RT = 15y - 9$ .

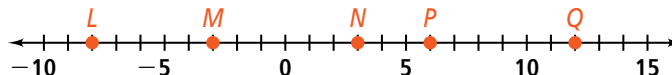
a. What is the value of  $y$ ?

b. Find  $RS$ ,  $ST$ , and  $RT$ .



See Problem 2.

Use the number line below for Exercises 15–18. Tell whether the segments are congruent.



15.  $\overline{LN}$  and  $\overline{MQ}$

16.  $\overline{MP}$  and  $\overline{NQ}$

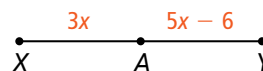
17.  $\overline{MN}$  and  $\overline{PQ}$

18.  $\overline{LP}$  and  $\overline{MQ}$

19. **Algebra**  $A$  is the midpoint of  $\overline{XY}$ .

a. Find  $XA$ .

b. Find  $AY$  and  $XY$ .



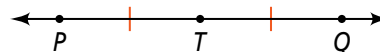
See Problem 4.

**Algebra** For Exercises 20–22, use the figure below. Find the value of  $PT$ .

20.  $PT = 5x + 3$  and  $TQ = 7x - 9$

21.  $PT = 4x - 6$  and  $TQ = 3x + 4$

22.  $PT = 7x - 24$  and  $TQ = 6x - 2$



## B Apply

On a number line, the coordinates of  $X$ ,  $Y$ ,  $Z$ , and  $W$  are  $-7$ ,  $-3$ ,  $1$ , and  $5$ , respectively. Find the lengths of the two segments. Then tell whether they are congruent.

23.  $\overline{XY}$  and  $\overline{ZW}$

24.  $\overline{ZX}$  and  $\overline{WY}$

25.  $\overline{YZ}$  and  $\overline{XW}$

Suppose the coordinate of  $A$  is  $0$ ,  $AR = 5$ , and  $AT = 7$ . What are the possible coordinates of the midpoint of the given segment?

26.  $\overline{AR}$

27.  $\overline{AT}$

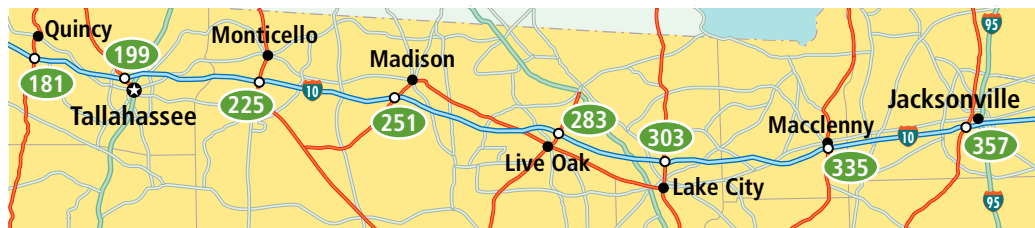
28.  $\overline{RT}$

29. Suppose point  $E$  has a coordinate of  $3$  and  $EG = 5$ . What are the possible coordinates of point  $G$ ?

- Visualization** Without using your ruler, sketch a segment with the given length. Use your ruler to see how well your sketch approximates the length provided.

30. 3 cm                      31. 3 in.                      32. 6 in.                      33. 10 cm                      34. 65 mm

- Think About a Plan** The numbers labeled on the map of Florida are mile markers. Assume that Route 10 between Quincy and Jacksonville is straight.



Suppose you drive at an average speed of 55 mi/h. How long will it take to get from Live Oak to Jacksonville?

- How can you use mile markers to find distances between points?
- How do average speed, distance, and time all relate to each other?

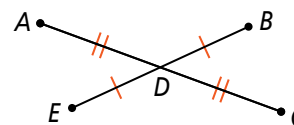
36. On a number line,  $A$  is at  $-2$  and  $B$  is at  $4$ . What is the coordinate of  $C$ , which is  $\frac{2}{3}$  of the way from  $A$  to  $B$ ?

- Error Analysis** Use the highway sign for Exercises 37 and 38.



37. A driver reads the highway sign and says, "It's 145 miles from Mitchell to Watertown." What error did the driver make? Explain.
38. Your friend reads the highway sign and says, "It's 71 miles to Watertown." Is your friend correct? Explain.

**Algebra** Use the diagram at the right for Exercises 39 and 40.



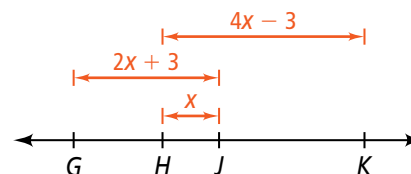
39. If  $AD = 12$  and  $AC = 4y - 36$ , find the value of  $y$ . Then find  $AC$  and  $DC$ .
40. If  $ED = x + 4$  and  $DB = 3x - 8$ , find  $ED$ ,  $DB$ , and  $EB$ .

- Writing** Suppose you know  $PQ$  and  $QR$ . Can you use the Segment Addition Postulate to find  $PR$ ? Explain.



42.  $C$  is the midpoint of  $\overline{AB}$ ,  $D$  is the midpoint of  $\overline{AC}$ ,  $E$  is the midpoint of  $\overline{AD}$ ,  $F$  is the midpoint of  $\overline{ED}$ ,  $G$  is the midpoint of  $\overline{EF}$ , and  $H$  is the midpoint of  $\overline{DB}$ . If  $DC = 16$ , what is  $GH$ ?

43. a. **Algebra** Use the diagram at the right. What algebraic expression represents  $GK$ ?
- b. If  $GK = 30$ , what are  $GH$  and  $JK$ ?

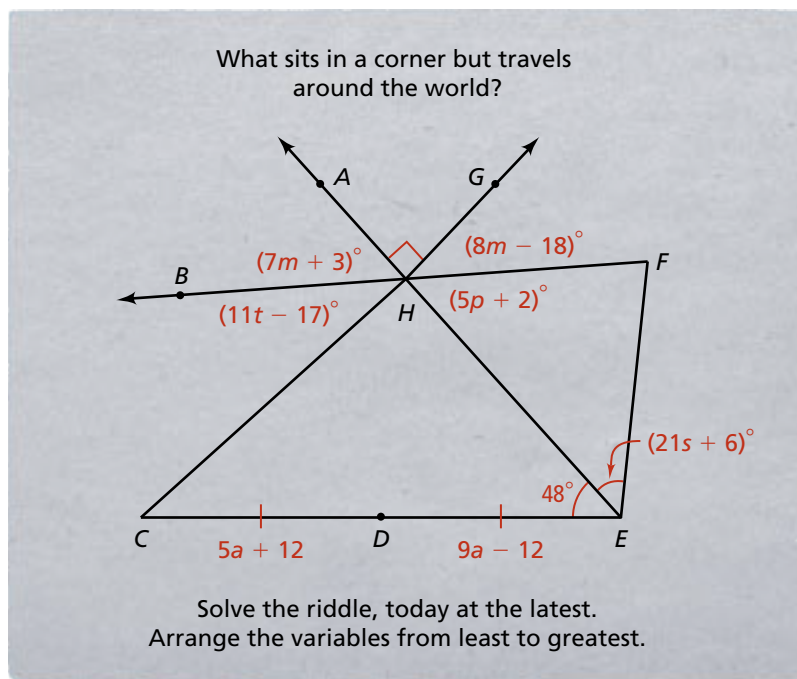




## Apply What You've Learned



Look back at the information on page 3 about the riddle Cameron found in an antique store. The page from the old riddle book is shown again below.



- What relationship between two segments can you state based on information in the diagram? How does the diagram show this relationship?
- Write and solve an equation to find the value of the variable  $a$ .
- How can you be sure you solved the equation in part (b) correctly?